

AP Calculus BC Summer Review Packet

Welcome to AP Calculus! This packet contains a set of problems that serve as a prerequisite for AP Calculus. These background skills are extremely important. Many times you will find that the Calculus is actually the easy part; it's the algebra that is difficult.

You are not required to do EVERY problem in this packet, but you are responsible for all content. I will check for completion of the packet within the first two days of school. You will have a day or two to ask questions on any problems or concepts you do not understand before you are tested on these concepts and skills. Be sure to circle any questions that you are unsure about so that it is easy for you to locate the problems you need clarification on during those first two days. Feel free to use extra paper for your work if you need more space.

There are many resources available to you to help with these topics. It might be helpful to start the habit of going online to my Calculus website (<http://goniocalc.weebly.com/>) to look for resources and answers. (Answer keys to homework and review packets are posted online throughout the year, so it's in your interest to know how to access this website!) You can find these resources and the **ANSWER KEY** to the summer packet under the AP Calculus Summer Packet Resources page.

Calculus can be challenging on its own, and with the pace that the BC course requires, it can be overwhelming if you do not keep up with the homework assigned. Start off on the right foot with this summer packet and get into the routine of completing the out of class work so that you are prepared to follow and understand the concepts in class.

I look forward to having you in class next year. If you have any questions over the summer, please feel free to contact me at brittanygonio@u-46.org.

Have a great summer!

Ms. Gonio

Simplifying Complex Fractions

Simplify each of the following.

$$1. \frac{x^3 - 9x}{x^2 - 7x + 12}$$

$$2. \frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$$

$$3. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

$$4. \frac{\frac{25}{a} - a}{5 + a}$$

Laws of Exponents

Write each of the following in the form ca^pb^q where c, p, and q are constants (numbers).

$$5. \frac{(2a^2)^2}{b}$$

$$6. \sqrt[3]{9ab^3}$$

$$7. \frac{ab - a}{b^2 - b}$$

$$8. \frac{a^{-1}}{(b^{-1})\sqrt{a}}$$

$$9. \left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$$

Laws of Logarithms

Simplify each of the following:

10. $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$

11. $3^{2 \log_3 5}$

12. $\log_{10} \frac{1}{10^x}$

Solving Exponential and Logarithmic Equations

Solve for x. (DO NOT USE A CALCULATOR.)

13. $5^{(x+1)} = 25$

14. $\frac{1}{3} = 3^{2x+2}$

15. $\log_2 x^2 = 3$

16. $\log_3 x^2 = 2 \log_3 4 - 4 \log_3 5$

Literal Equations

Solve for the indicated variables.

17. $V = 2(ab + bc + ca)$, for a

18. $A = P + \pi rP$, for P

19. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$, for x

Real Solutions

Find all real solutions.

20. $x^4 - 1 = 0$

21. $x^6 - 16x^4 = 0$

22. $4x^3 - 8x^2 - 25x + 50 = 0$

Solving Equations

Solve the equations for x .

23. $4x^2 + 12x + 3 = 0$

24. $2x + 1 = \frac{5}{x+2}$

25. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Polynomial Division

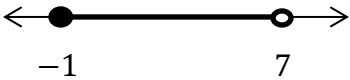
26. $(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$

27. $(x^6 + 2x^4 + 6x - 9) \div (x^3 + 3)$

28. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

Interval Notation

29. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$(-\infty, 8]$	
		

Solving Inequalities

Solve the inequalities. Write the solution in interval notation.

30. $x^2 + 2x - 3 \leq 0$

31. $\frac{2x-1}{3x-2} \leq 1$

32. $\frac{2}{2x+3} > \frac{2}{x-5}$

Solving Equations with Absolute Value

Solve for x. Give the solution for inequalities in interval notation.

33. $|-x + 4| \leq 1$

34. $|5x - 2| = 8$

35. $|2x + 1| > 3$

Functions

To evaluate a function for a given value, simply plug the value into the function for x .

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read “ f of g of x ” Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned}f(g(x)) &= f(x - 4) \\ &= 2(x - 4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33\end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each of the following.

36. $f(2) =$ _____ 37. $g(-3) =$ _____ 38. $f(t + 1) =$ _____

39. $f(g(-2)) =$ _____ 40. $g(f(m + 2)) =$ _____

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each of the following.

41. $h(f(-2)) =$ _____

42. $f(g(x - 1)) =$ _____

43. $g(h(x^3)) =$ _____

Intercepts and Points of Intersection

To find the x-intercepts, let $y = 0$ in your equation and solve.

To find the y-intercepts, let $x = 0$ in your equation and solve.

Example: $y = x^2 - 2x - 3$

x - int. (Let $y = 0$)

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$x = -1 \text{ or } x = 3$$

x - intercepts $(-1, 0)$ and $(3, 0)$

y - int. (Let $x = 0$)

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

y - intercept $(0, -3)$

Find the x and y intercepts of each.

44. $y = 2x - 5$

45. $y = x^2 + x - 2$

46. $y = \sqrt{16 - x^2}$

Domain and Range

Find the domain and range of each function. Write your answer in interval notation.

47. $f(x) = x^2 - 5$

48. $f(x) = 3 \sin x$

49. $f(x) = \frac{2}{x-1}$

Systems

Use substitution or elimination method to solve the system of equations.

Example:

$$x^2 + y - 16x + 39 = 0$$

$$x^2 - y^2 - 9 = 0$$

Elimination Method

$$2x^2 - 16x + 30 = 0$$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5) = 0$$

$$x = 3 \text{ and } x = 5$$

Plug $x=3$ and $x = 5$ into one original

$$3^2 - y^2 - 9 = 0 \quad 5^2 - y^2 - 9 = 0$$

$$-y^2 = 0 \quad 16 = y^2$$

$$y = 0 \quad y = \pm 4$$

Points of Intersection $(5, 4)$, $(5, -4)$ and $(3, 0)$

Substitution Method

Solve one equation for one variable.

$$y^2 = -x^2 + 16x - 39 \quad (\text{1st equation solved for } y)$$

$$x^2 - (-x^2 + 16x - 39) - 9 = 0 \quad \text{Plug what } y^2 \text{ is equal to into second equation.}$$

$$2x^2 - 16x + 30 = 0 \quad (\text{The rest is the same as previous example})$$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5) = 0$$

$$x = 3 \text{ or } x = 5$$

Find the point(s) of intersection of the graphs for the given equations.

$$50. \begin{cases} x + y = 8 \\ 4x - y = 7 \end{cases}$$

$$51. \begin{cases} x^2 + y = 6 \\ x + y = 4 \end{cases}$$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new “ y ” value.

Example:

$$f(x) = \sqrt[3]{x+1} \quad \text{Rewrite } f(x) \text{ as } y$$

$$y = \sqrt[3]{x+1} \quad \text{Switch } x \text{ and } y$$

$$x = \sqrt[3]{y+1} \quad \text{Solve for your new } y$$

$$(x)^3 = (\sqrt[3]{y+1})^3 \quad \text{Cube both sides}$$

$$x^3 = y+1 \quad \text{Simplify}$$

$$y = x^3 - 1 \quad \text{Solve for } y$$

$$f^{-1}(x) = x^3 - 1 \quad \text{Rewrite in inverse notation}$$

Find the inverse for each function.

52. $f(x) = 2x + 3$

53. $f(x) = \frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
 $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ and $g(x) = 4x+9$ **show $f(x)$ and $g(x)$ are inverses of each other.**

$$\begin{aligned}g(f(x)) &= 4\left(\frac{x-9}{4}\right) + 9 \\ &= x - 9 + 9 \\ &= x\end{aligned}$$

$$\begin{aligned}f(g(x)) &= \frac{(4x+9)-9}{4} \\ &= \frac{4x+9-9}{4} \\ &= \frac{4x}{4} \\ &= x\end{aligned}$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove f and g are inverses of each other.

54. $f(x) = \frac{x^3}{2}$ $g(x) = \sqrt[3]{2x}$

55. $f(x) = 9 - x^2$ $g(x) = \sqrt{9 - x}$

Vertical Asymptotes

Determine the vertical asymptotes for each function. Set the denominator equal to zero to find the x-value for which the function is defined. This will be the vertical asymptote.

$$56. f(x) = \frac{1}{x^2}$$

$$57. f(x) = \frac{x^2}{x^2-4}$$

$$58. f(x) = \frac{2+x}{x^2(1-x)}$$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I: Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II: Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III: Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all horizontal asymptotes.

$$59. f(x) = \frac{x^2-2x+1}{x^3+x-7}$$

$$60. f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$$

$$61. f(x) = \frac{4x^5}{x^2-7}$$

Equation of a Line

Slope Intercept Form: $y = mx + b$

Vertical Line: $x = c$ (slope is undefined)

Point-slope Form: $y - y_1 = m(x - x_1)$

Horizontal Line: $y = c$ (slope is 0)

62. Use slope-intercept form to find the equation of the line having slope of 3 and a y-intercept of 5.

63. Determine the equation of a line passing through the point $(5, -3)$ with an undefined slope.

64. Determine the equation of a line passing through the point $(-4, 2)$ with a slope of 0.

65. Use point-slope form to find the equation of a line passing through the point $(0, 5)$ with a slope of $2/3$.

66. Find the equation of a line passing through the point $(6, 8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

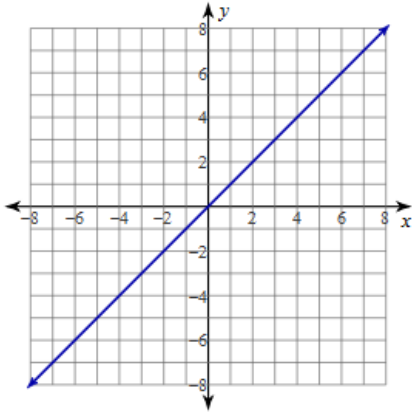
67. Find the equation of a line passing through points $(-3, 6)$ and $(1, 2)$.

68. Find the equation of a line with an x-intercept of $(2, 0)$ and a y-intercept $(0, 3)$.

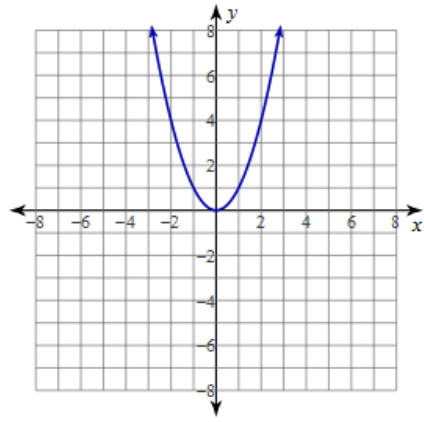
Parent Functions

For 69 – 78, identify the parent function associated with each graph.

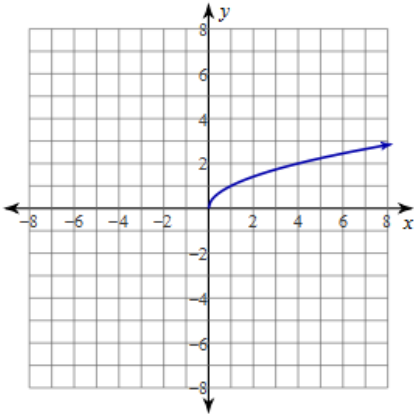
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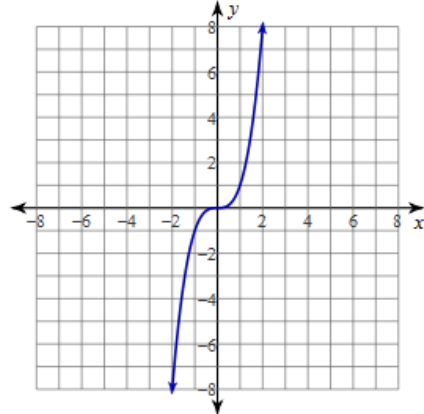
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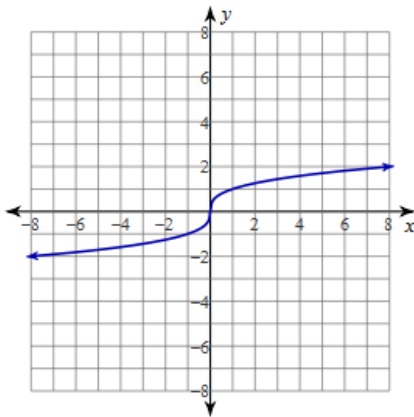
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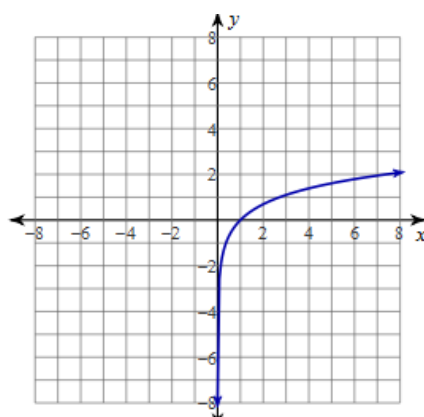
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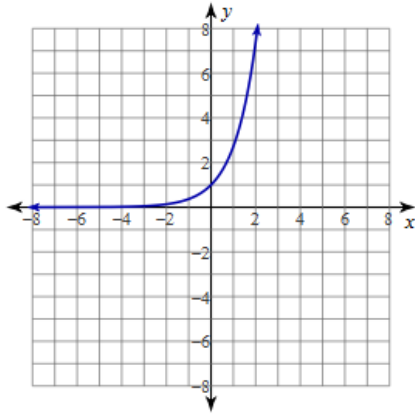
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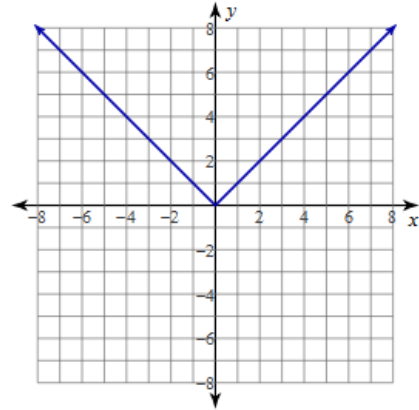
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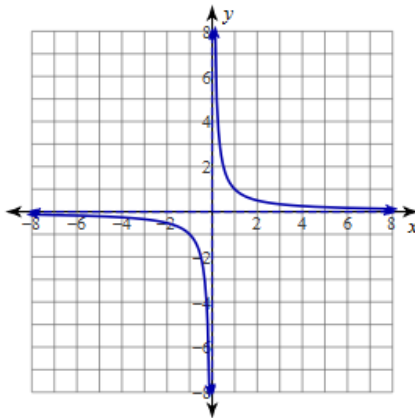
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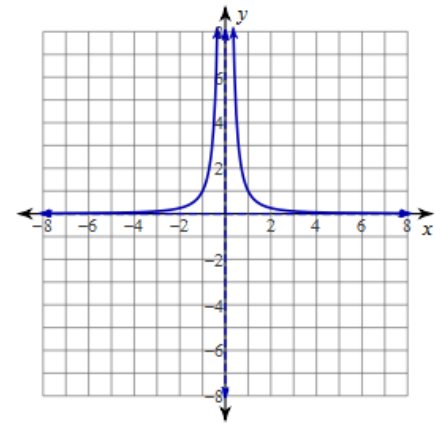
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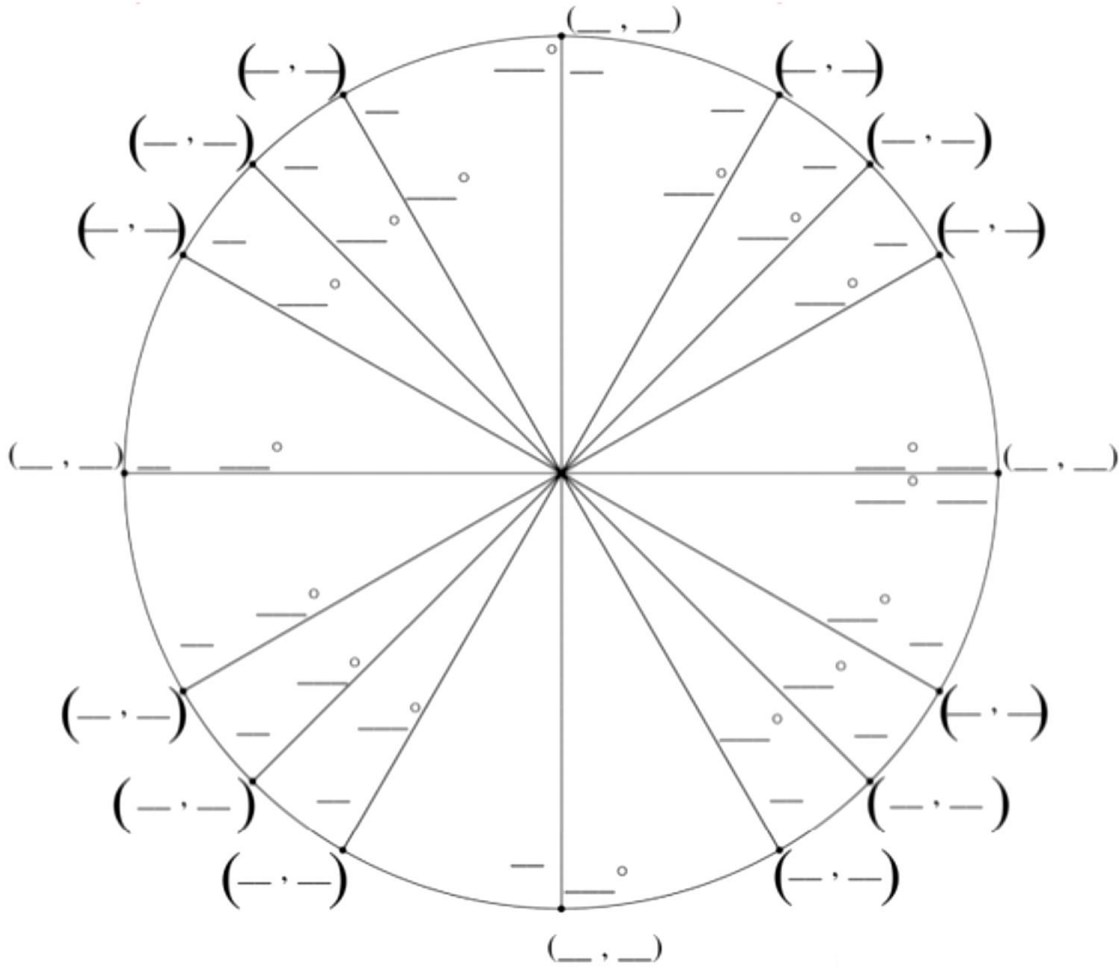


78.



Unit Circle

79. Identify all parts of the unit circle, including degree, radian, and coordinates of each point.



Without using a calculator, evaluate the following.

80. a) $\sin 180^\circ$

b) $\cos 270^\circ$

c) $\sin \pi$

d) $\cos(-\pi)$

e) $\sin \frac{5\pi}{4}$

f) $\cos \frac{9\pi}{4}$

g) $\tan \frac{7\pi}{6}$

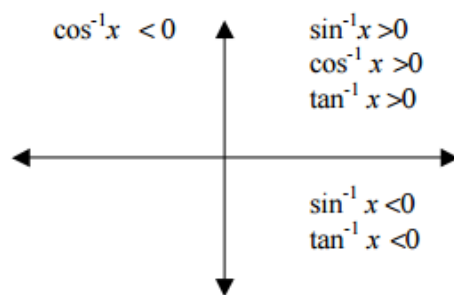
Inverse Trigonometric Functions

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x)$$

$$\sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.

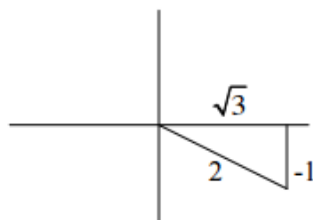


Example:

Express the value of “y” in radians.

$$y = \arctan \frac{-1}{\sqrt{3}}$$

Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$$\frac{-\pi}{2} < y < \frac{\pi}{2}$$

$$\text{Answer: } y = -\frac{\pi}{6}$$

For each of the following, find the value in radians.

81. $y = \sin^{-1} \frac{-\sqrt{3}}{2}$

82. $y = \arccos(-1)$

83. $y = \tan^{-1}(-1)$

84. $y = \cos^{-1} \left(\sin \left(-\frac{\pi}{4} \right) \right)$

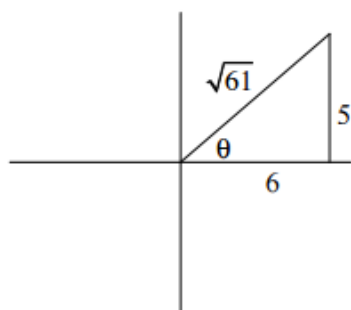
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

85. $\tan\left(\arccos\frac{2}{3}\right)$

86. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

87. $\sin\left(\arcsin\frac{7}{8}\right)$

Trigonometric Equations

Solve each of the equations for $0 \leq x < 2\pi$. Isolate the variable and find all the solutions within the given domain. Remember to double the domain when solving for a double angle. Use trig identities, or rewrite the trig functions using substitution, if needed.

88. $\sin x = -\frac{1}{2}$

89. $2 \cos x = \sqrt{3}$

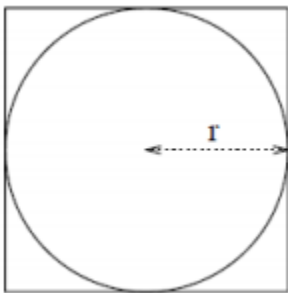
$$90. \sin^2 x = \frac{1}{2}$$

$$91. \sin 2x = -\frac{\sqrt{3}}{2}$$

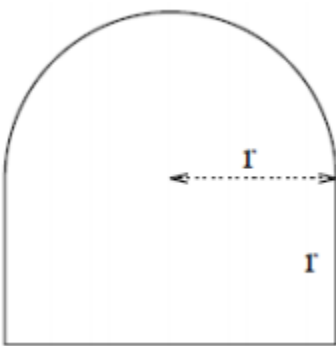
$$92. 2 \cos^2 x - 1 - \cos x = 0$$

$$93. 4 \cos^2 x - 3 = 0$$

94. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture below.



95. Find the formula for the perimeter of the window of the shape in the picture below.



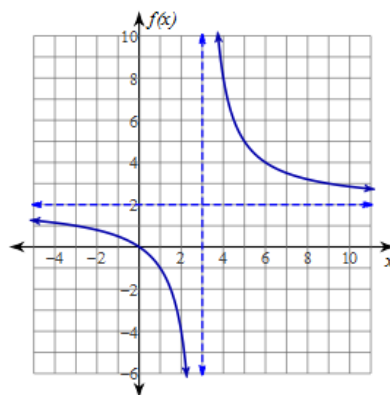
96. A water tank has the shape of a cone (where $V = \frac{\pi}{3} r^2 h$). The tank is 10 m high and has a radius of 3 m at the top. When the water is 5 m deep (in the middle of the tank) what is the surface area of the top of the water?

97. Two cars start moving from the same point. One travels south at 100 km/hr, the other west at 50 km/hr. How far apart are they two hours later?

98. A kite is 100 m above ground. If there is 200 m of string connecting the kite to the horizontal, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)

Limits

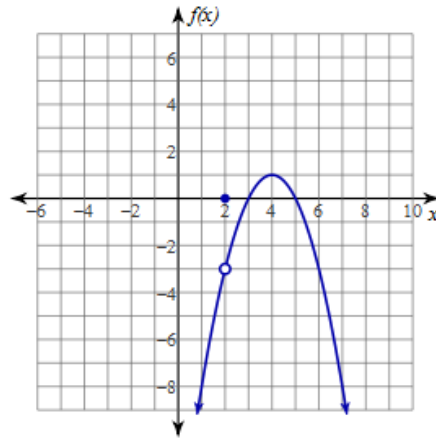
Using the graphs, find the following limits.



99. $\lim_{x \rightarrow 3^+} f(x) =$

100. $\lim_{x \rightarrow \infty} f(x) =$

101. $\lim_{x \rightarrow 3} f(x) =$



102. $\lim_{x \rightarrow 2} f(x) =$

103. $f(2) =$

Find $\frac{f(x+h)-f(x)}{h}$ for the given function f .

104. $f(x) = 2x + 3$

105. $f(x) = 3x^2 - x + 5$

106. $f(x) = 5 - 2x$